

# TOPCOM User Manual

version 1.2.0

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## 1 Introductory Remark

The new version 1.2.0 now supports checkpointing for enumerations of triangulations, circuits, and cocircuits.

Versions starting at 1.1.0 use vastly improved enumeration methods for

- enumeration of all triangulations,
- enumeration of circuits from points,
- enumeration of cocircuits from points.

Moreover, lock-free multithreading is used on demand for enumeration and/or symmetry processing.

For really large problem instances (co-dimension above 13), the exploration-by-flipping of the regular component of the flip-graph in TOPCOM's `points2(n)triangs` takes a lot of memory. The exploration of the (in general smaller) subregular component of the flip graph can be done with less memory consumption by using MPTOPCOM by Jordan, Joswig, and Kastner (<https://polymake.org/doku.php/mptopcom>). The enumeration of *all* triangulations, however, can be done with similar memory consumption now by the TOPCOM client `points2(n)alltriangs`. This client, for the time being, is the fastest method in TOPCOM to enumerate all triangulations of point configurations.

## 2 What is TOPCOM?

TOPCOM is a collection of clients to compute *Triangulations Of Point Configurations and Oriented Matroids*, resp.

The algorithms use only combinatorial data of the point configuration as is given by its oriented matroid. Some basic commands for computing and manipulating oriented matroids can also be accessed by the user.

## 3 How do I use TOPCOM?

All programs read the input from `stdin` and write the result to `stdout` so that you can pipe the results to the next command.

A point configuration is given by a matrix (enclosed in square brackets) whose columns (enclosed in square brackets) are the homogeneous coordinates (seperated by commas) of the points in the configuration. A square could be specified as follows.

```
[[0,0,1],[0,1,1],[1,0,1],[1,1,1]]
```

You may specify generators of the combinatorial symmetry of a point configuration as permutations of the vertex numbers. The symmetry generators of the square read as follows (observe that the count starts at 0):

```
[[3,2,1,0],[2,3,0,1],[0,2,1,3]]
```

Some algorithms also read in a given seed triangulation given by a set of sets of numbers, where elements of a set are enclosed in curly brackets and separated by commas. For example:

```
{{0,1,2},{1,2,3}}
```

If you want to give a seed but no symmetries, specify `[]` for the symmetries.

Finally, some algorithms observe prescribed symmetries for the resulting triangulations. To generate only triangulations of the square symmetric w.r.t. the diagonal of the first quadrant, add the following line:

```
[[3,1,2,0]]
```

If you want to prescribe symmetries but no seed, specify `{}` for the seed.

## 4 Commands

The following commands are provided:

`points2prettyprint` (New.) Displays the point and their symmetry generators in a more readable form.

`points2chiro` Computes the chirotope of a point configuration.

`chiro2dual` Computes the dual of a chirotope.

`chiro2circuits` Computes the circuits of a chirotope.

`points2circuits` Dto. for point configurations (using a faster method).

`chiro2cocircuits` Computes the cocircuits of a chirotope.

`points2cocircuits` Dto. for point configurations (using a faster method).

`cocircuits2facets` Computes the facets of a set of cocircuits.

`points2facets` Computes the facets of a point configuration.

`points2gale` Computes a Gale transform of a point configuration.

`chiro2circuits` Computes the circuits of a point configuration.

`chiro2cocircuits` Computes the cocircuits of a point configuration.

`points2facets` Computes the facets of a point configuration.

`points2nflips` Computes the number of flips of a point configurations and the seed triangulation.

`points2flips` Computes all flips of a point configurations and the seed triangulation.

`chiro2placingtriang` Computes the placing triangulation of a chirotope given by the numbering of the elements.

`points2placingtriang` Dto. for point configurations.

`chiro2finetriang` Computes a fine (i.e., using all vertices) triangulation by placing and pushing.

`points2finetriang` Dto. for point configurations.

`chiro2triangs` Computes all triangulations of a chirotope that are connected by bistellar flips to the seed, which is a regular triangulation if no seed is given in the input file.

`points2triangs` Dto. for point configurations.

`chiro2ntriangs` Computes the number of all triangulations of a chirotope that are connected by bistellar flips to the seed, which is a regular triangulation if no seed is given in the input file.

`points2ntriangs` Dto. for point configurations.

`chiro2finetriangs` Computes all fine triangulations (the ones that use all the points sometimes called “full”) of a chirotope that are connected by bistellar flips to a fine seed triangulation.

`points2finetriangs` Dto. for point configurations.

`chiro2nfinetriangs` Computes the number of all fine triangulations of a chirotope that are connected by bistellar flips to a fine seed triangulation.

`points2nfinetriangs` Dto. for point configurations.

`chiro2alltriangs` Computes all triangulations of a chirotope.

`points2alltriangs` Dto. for point configurations.

`chiro2nalltriangs` Computes the number of all triangulations of a chirotope.

`points2nalltriangs` Dto. for point configurations.

`chiro2allfinetriangs` Computes all fine triangulations (sometimes called “full”) of a chirotope.

`points2allfinetriangs` Dto. for point configurations.

`chiro2nallfinetriangs` Computes the number of all fine triangulations of a chirotope.

`points2nallfinetriangs` Dto. for point configurations.

`chiro2mintriang` Computes a triangulation of a chirotope with a minimum number of simplices.

`points2mintriang` Dto. for point configurations.

`B_S n` Computes the vertices and symmetry generators of the permutation polytope of the symmetric group of degree  $n$ , also known as the Birkhoff polytope.

`B_A n` Computes the vertices and symmetry generators of the permutation polytope of the alternating group of degree  $n$ , also known as the even Birkhoff polytope.

`B_D n` Computes the vertices and symmetry generators of the permutation polytope of the dihedral group of degree  $n$ .

`B_S_center n` Computes `B_S n` with an additional center point.

`B_A_center n` Computes `B_A n` with an additional center point.

`B_D_center n` Computes `B_D n` with an additional center point.

`cube d` Computes the vertices and symmetry generators of a  $d$ -cube.

`cyclic n d` Computes the vertices and symmetry generators of the cyclic  $d$ -polytope with  $n$  vertices.

`cross d` Computes the vertices and symmetry generators of the  $d$ -dimensional cross-polytope.

`kDn k n` Computes the vertices and symmetry generators of the  $k$ -times dilated  $n$ -simplex with all interior lattice points.

`Dnxk n k` Computes the vertices and symmetry generators of  $k$  copies of the  $n$ -simplex.

`lattice n m` Computes the  $nm$  two-dimensional lattice points with non-negative coordinates at most  $(n-1, m-1)$  and their symmetry generators.

`hypersimplex d k [ $\ell$ ]` Computes the vertices and symmetry generators of the  $k$ -th hypersimplex in dimension  $d$ . A third parameter makes it the  $S$ -hypersimplex with coordinate sums equal to  $k$  or  $\ell$ .

`santos_triangu` Computes the point configuration, the symmetry, and the Santos triangulation (without flips).

## 5 Command Line Options

The following command line options are supported. Note that not all options are sensible for all clients.

## Options concerning input/output from files

`-i [filename]`: read input from [filename] instead of stdin.

## Options concerning output of information

`-h` or `--help` Print a usage message.

`-d` Debug.

`-v` Verbose.

`--reportfrequency [n]` In enumerations, report intermediate results after (at least) [n]th discovered nodes.

`--heights` Output a height vector for every regular triangulation (implies `--regular`).

`--flips` Output all flips in terms of IDs of adjacent triangulations. (Can be used to generate the flip graph.)

`--asy` Write asymptote graphics commands into file (in rank-3 triangulations, points are drawn as well). The graphics contains a view of the point configuration (only in rank 3), the enumeration tree with a classification of enumeration nodes into solutions, non-canonical nodes, deadends, and early detected deadends, as well a statistics file showing a histogram of enumeration node types. The output file has to be processed by the computer graphics compiler asy (<https://asymptote.sourceforge.io>) using the asy-library `Combinatorial_Geometry.asy` and the  $\text{\LaTeX}$ -macroses in `triangbook_macroses.sty` inside `share/asy/`.

## Options for checking input

`--checktriang` Check seed triangulation.

## Options for reporting properties of discovered triangulations:

`--flipdeficiency` Check triangulations for flip deficiency during flip-graph exploration.

`--findregular [k]` (New form.) Check every  $k$ -th triangulation for regularity and stop if a regular one is found during flip-graph exploration.

## Options concerning which triangulations are output (no influence on flip-graph exploration)

- `--noorbitcount` Only count symmetry classes, not the total number.
- `--cardinality [k]` Count/output only triangulations with exactly  $k$  simplices.
- `--maxcardinality [k]` Count/output only triangulations with at most  $k$  simplices.
- `--unimodular` (New.) Output unimodular triangulations only; while this does not reduce the effort of flip graph exploration, since unimodular triangulations are in general not connected by themselves, it does reduce the effort of extension graph exploration like in `points2nalltriangs`.
- `--requirepoint [k]` Count/output only triangulations with simplices containing point  $k$  (currently only supported by `points2(n)alltriangs`). Note that in this case only symmetries stabilizing that point are considered.
- `--nonregular` Output non-regular triangulations only; note that this does not reduce the effort of flip-graph exploration, since non-regular triangulations are in general not connected by themselves.

## Options concerning which triangulations are explored

- `--regular` Search for regular triangulations only (checked liftings are w.r.t. the last homogeneous coordinate, e.g., last coordinates all ones is fine); note that this may reduce the effort of flip-graph exploration, since regular triangulations are connected by themselves.
- `--regularearly` Search for regular partial triangulations only; note that this may reduce the effort of extension-graph exploration, since no non-regular partial triangulation can be completed to a regular triangulation.
- `--noinsertion` Never flip-in a point that is unused in the seed triangulation.
- `--reducepoints` Try to greedily minimize the number of vertices used while flipping; keep a global upper bound on the current minimal number of vertices and do not accept triangulations with more vertices.
- `--keepcard` Never change the cardinality of triangulations by flipping.

## Options concerning symmetries

- `--affinesymmetries` (New form.) Assume that the symmetries are affine, in particular, that they conserve regularity.
- `--isometricsymmetries` (New.) Assume that the symmetries are isometric, in particular, that they preserve volume.
- `--nosymmetries` Ignore the symmetries.
- `--triangsymmetries` (New.) Observe the prescribed symmetries for triangulations in the result.
- `--notriangsymmetries` (New; currently the default.) Ignore the prescribed symmetries for triangulations in the result (option supported for backwards-compatibility).

## Options controlling the internals of the clients

- `--memopt` Save memory by using caching techniques.
- `--iterativedfs` (New.) Use an iterative version of the depth-first-search enumeration algorithms (avoids potential call-stack overflows on small machines).
- `--recursivedfs` (New.) Use a recursive version of the depth-first-search enumeration algorithms (default).
- `--usegkz` (New.) Use GKZ vectors as a finger print in symmetry handling (only for points with isometric symmetries).
- `--usenaivesymmetries` (New.) Use naive full traversal of all symmetries for symmetry handling.
- `--useswitchtables` (New.) Use Jordan-Joswig-Kastner switch tables for symmetry handling.
- `--usesymmetrytables` (New.) Use tables of classified symmetries for symmetry handling. Obsolete, since slower than the other options.
- `--symtables [n]` (New.) Use [n] symtables for preprocessing symmetries. Obsolete, since slower than the other options.
- `--preprocesschiro` (New.) Preprocess the chirotope (default for `points2[n] alltriangs`).
- `--preprocesspoints` (New.) Heuristically transform points (only relevant for (co)circuit enumeration).



- `--simpidxsymmetries` (New.) Preprocess a representation of the symmetry group on simplex indices (only relevant for triangulation enumeration).
- `--userandomorder` (New.) Sort simplices in preprocessed index table randomly (only for points with isometric symmetries).
- `--usevolumeorder` (New.) Sort simplices in preprocessed index table by volume (only for points with isometric symmetries).
- `--usevolumes` (New.) Use volumes to check extendability of partial triangulations (only for points with isometric symmetries).
- `--fullextensioncheck` (New.) Put more effort in the check of extendability of a partial triangulation.
- `--noextensioncheck` (New.) Skip the check of extendability of a partial triangulation.
- `--extensioncheckfirst` (New.) Check extendability prior to symmetry.
- `--preprocesspoints` Preprocess the coordinate matrix of the points (slightly useful for (co-)circuit enumeration)
- `--chirocache`  $[n]$  Set the chirotope cache to  $n$  elements.
- `--localcache`  $[n]$  Set the cache for local operations.
- `--qsopt_ex` Use QSopt\_ex for regularity checks (not thread-safe).
- `--soplex` Use soplex for regularity checks (requires separate installation of soplex).

## Options concerning multi-threading

- `--parallelenumeration` Use multiple threads for enumeration.
- `--workbuffercontrol` (Currently the default.) Control the interrupt of workers by size of the current workbuffer.
- `--noworkbuffercontrol` Control the interrupt of workers by node budgets.
- `--parallelsymmetries` Use multiple threads only locally for symmetry checks.
- `--threads`  $[n]$  Use  $[n]$  threads (if possible).

`--minnodebudget [n]` Let each thread process at least `[n]` nodes (to avoid multi-threading overhead).  
`--maxnodebudget [n]` Let each thread process at most `[n]` nodes (to avoid thread starving).  
`--scalenodebudget [n]` Scale the default node budget by `[n]` percent (`n` integer)  
`--minworkbuffer [n]` (Currently unused.) Try to keep the work buffer above `[n]` nodes (to balance overhead and thread starving).  
`--maxworkbuffer [n]` (Currently unused.) Try to keep the work buffer below `[n]` node (to balance overhead and thread starving).

## Options for warm starts from previous calculations

These options currently work for the clients `points2(n)triangs`, `chiro2(n)triangs`, `points2(n)alltriangs`, `chiro2(n)alltriangs`, and `points2(n)(co)circuits`.

`--dump` Write intermediate results into a file; a symbolic link with the basename will be placed pointing to the most recent actual dump file.  
`--dumprotations [k]` Dump into  $k$  different rotating files indicated by numerical postfixes.  
`--dumpfile [dumpfilename]` Write intermediate results into files with basename `dumpfilename` (default: `TOPCOM.dump`); a symbolic link with the basename will be placed pointing to the most recent actual dump file.  
`--dumpfrequency [k]` Dump the results at each  $k$ th “object” (what object means, depends on the kind of enumeration)  
`--read` Read intermediate results from a file.  
`--readfile [readfilename]` Read intermediate results from file `dumpfilename` (default: `TOPCOM.dump`).

## 6 Examples

In the subdirectory `examples` you find some example inputs for TOPCOM routines.

If you want to see the input in a prettier way, then you can type the following:

```
points2prettyprint < cube_3.dat
```

It prints the matrix of point coordinates and the symmetry generators in tabular form.

Or:

```
points2chiro < lattice_3_3.dat
```

outputs the sign string of the chirotope of the sub-lattice of integer points  $(i, j)$  with  $i, j = 0, 1, 2$ .

```
points2chiro < lattice_3_3.dat | chiro2ntriangs
```

or

```
points2ntriangs < lattice_3_3.dat
```

yields the number of triangulations that are connected to the regular ones by bistellar flips.

```
points2ntriangs --regular --affinesymmetries < moae.dat
```

counts all regular triangulations of the “mother of all examples”, two nested triangles in the plane.

```
points2nalltriangs < lattice_3_3.dat
```

yields the number of all triangulations via a new algorithm *symmetric lex-subset reverse-search* on partial triangulations. It is mostly faster than flip-graph exploration and much faster than in older versions of TOPCOM; moreover, it does not take a lot of memory.

The example `r12.chiro` is the chirotope of the oriented matroid  $R12$  with disconnected realization space, constructed by Jürgen Richter-Gebert. If you want to compute, e.g., a placing triangulation of  $R12$  then type

```
chiro2placingtriang < r12.chiro
```

The facets of a 4-cube can be computed by

```
cube 4 | points2facets
```

but be aware of the fact that this is not an efficient way of computing facets of a point configuration. It is, however, numerically stable because rational arithmetics is used.

Most notably, you can check the Santos triangulation by

```
santos_triang | points2nflips -v --memopt --checktriang
```

Recall that the options mean:

`-v` verbose;

`--memopt` save memory;

`--checktriang` check seed triangulation.

A new result as of version 1.0.10 is that TOPCOM can now enumerate *all* triangulations of larger point configurations like the 4-cube in reasonable time. The speed depends on the computational environment and the options:

```
cube 4 | points2nalltriangs -v --parallelenumeration
```

uses verbose output and multi-threaded symmetry processing with load-balancing based on work buffer control, i.e., the workers stop and push the open nodes into the job list whenever the master's job list contains fewer elements than there are workers.

Since not every point configuration can be represented in rational coordinates, it is useful to enumerate triangulations from the chirotope data. This way you can compute, e.g., all triangulations of the regular icosahedron:

```
chiro2nalltriangs -i icosahedron.chiro -v
```

Note the newly supported alternative syntax `-i [filename]` for the specification of the input file.

The enumeration of circuits and cocircuits up to symmetry is now a non-trivial part of TOPCOM. For example:

```
cube 4 | points2ncircuits -v --parallelenumeration
```

enumerates the symmetry classed of all the circuits of the 4-cube.

Moreover, the clients to explore the flip graph for points can now use the GKZ vector as a fingerprint to detect known triangulation classes in case all symmetries are isometric. This is faster, but needs some more memory for storing all the known GKZ vectors. Moreover, a basic multi-threaded symmetry check can be used. You can activate all of this by:

```
cube 4 | points2ntriangs --parallelsymmetries \  
--usegkz --isometricsymmetries
```

As an example for the graphics output the 2-by-3 lattice in dimension two is presented. Graphics output is supported by the enumeration of all triangulations, circuits and co-circuits. The files `Combinatorial_Geometry.asy` and `triangbook_macroses.sty` (to be found under `share/asy` in the TOPCOM home directory) must be present in the working directory where `TOPCOM_graphics.asy` is compiled via `asy`:

```
lattice 2 3 | points2nalltriangs -v --parallelenumeration --asy  
asy TOPCOM_graphics.asy
```

The point configuration can be found in `TOPCOM_points.pdf` (see Figure figure1). The enumeration tree can be found in `TOPCOM_tree_graph.pdf` (see Figure figure2).

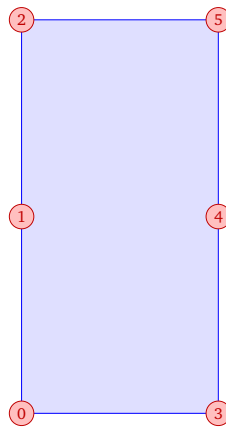


Figure 1: The points

Here, a *clover icon* means a triangulation in a new symmetry class, a *recycling icon* means a node in an old symmetry class, a *stop-sign icon* means a deadend with no

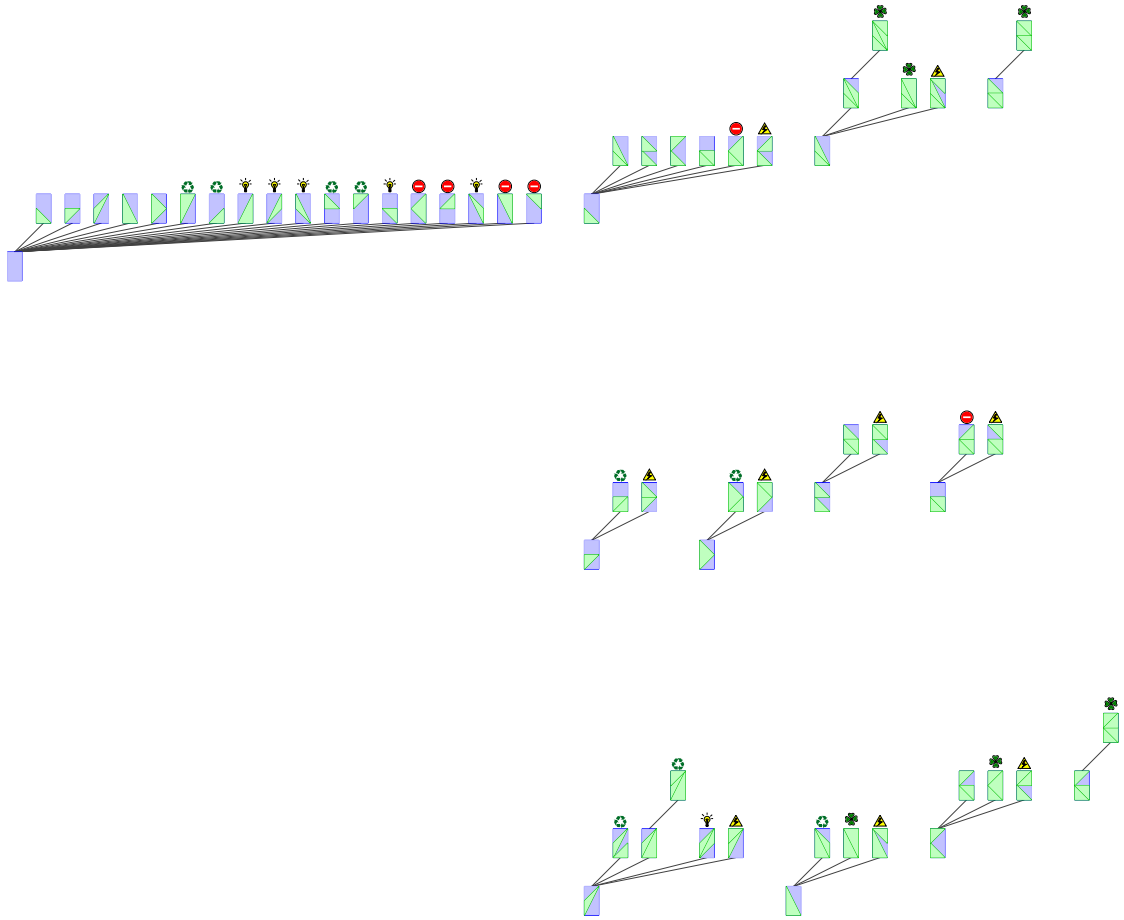


Figure 2: The enumeration tree with classified nodes – here with three threads and workbuffercontrol

possible admissible lex-larger extension, a *bulb icon* means an early detected deadend with an uncoverable interior facet, and a *lightning-sign icon* means that no further extension option can prevent an uncoverable interior facet. Statistics can be found in TOPCOM\_statistics.pdf (see Figure figure3).

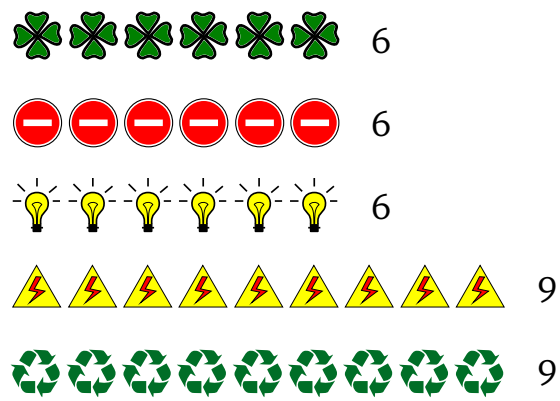


Figure 3: The statistics of classified nodes